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Potential of Global Positioning System (GPS) to measure frequencies of oscillations of engineering structures

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Abstract

Global Positioning System (GPS) has been successfully used to measure displacements of oscillating flexible civil engineering structures such as long suspension bridges and high-rise buildings, and to derive their modal frequencies, usually up to 1 Hz, but there is evidence that these limits can be exceeded using high frequency GPS receivers. Based on systematic experiments in computer controlled oscillations with one- and three-degrees of freedom we investigated the potential of GPS, first to record higher oscillation frequencies, at least up to 4 Hz at the minimum resolution level of this instrument for kinematic applications ($\geqslant$5 mm), and second, to identify more than one dominant frequency. Data were processed using least squares-based spectral analysis and wavelet techniques which permit to analyze entire time series, even those of too short duration or those characterized by gaps, in both the frequency and the time domain.

The ability of GPS to accurately measure frequencies of oscillations of relatively rigid (modal frequencies 1–4 Hz) civil engineering structures is demonstrated in the cases of two bridges.

The outcome of this study is that GPS is suitable for the identification of dynamic characteristics of even relatively rigid (modal frequencies up to 4 Hz) civil engineering structures excited by various loads (wind, traffic, earthquakes, etc.) if displacements are above the uncertainty level of the method ($\geqslant$5 mm). Structural health monitoring of a wide range of structures appears therefore a promising field of application of GPS.

1. Introduction

In the last years, after the pioneering study of Lovse et al. [1], Roberts et al. [2] and others who used Global Positioning System (GPS) to measure oscillations of major engineering structures, and of Celebi and Sanli [3] and of others who assessed the potential of this technique using independent evidence, GPS appears as a promising solution for the study and monitoring of semi-static and dynamic displacements and deformations of large flexible engineering structures, mostly high-rise buildings and large suspension buildings, i.e. structures with primary modal frequencies usually <1 Hz [4–8]. Furthermore, GPS proved an essential tool for the construction of modern sky-scrappers, where chaotic patterned oscillations are observed [9], due to the

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wind loads, temperature variations [10] and crane-induced dynamic loads, which make the precise definition of the static vertical axes almost impossible. For this reason a system of sensors, mainly GPS, is used to define the time-dependent geometry of such structures during their construction [11].

The potential of GPS to monitor displacements above the threshold of 5–10 mm has been documented mostly on the basis of experiments in which the known oscillatory movement of a certain device was compared with that recorded by a GPS receiver mounted on this device in comparison to another nearby GPS receiver fixed on stable ground. Such experiments revealed that GPS can identify oscillations with a period up to 1–2 Hz and with amplitude higher than 5–10 mm [4,5,11–14] and that there is a tendency for the signal-to-noise ratio to increase with oscillation amplitude and decrease with oscillation frequency [5,14].

Since, however, the measurement rate of modern receivers is 10–20 Hz with a tendency to increase, a reasonable question arising is whether GPS can be used in the monitoring of higher-frequency (>1–2 Hz) oscillations and overcome the problem of errors induced by computations of displacements from accelerometer records [15]. Recent studies were mainly focusing on the determination of the accuracy and range of displacements measured by GPS [4,12] and it was in particular shown that GPS can measure displacements in a much broader range of oscillations, up to 4 Hz [14]. Still, the problem of identification of oscillation frequencies from GPS data has been examined either for large displacement signals [16] or has not been examined in full detail [7,8,10,14].

This leaves much ground to systematically investigate the following problems: (1) what is the range and level of accuracy of modal frequencies determined with GPS? (2) Does this accuracy depend on the frequency of recorded oscillations? (3) Can variations in modal frequencies and multiple modal frequencies in a single displacement record be identified?

In an effort to answer these questions, we made a large number of experiments to test the tolerance of GPS using an experimental technique not very different from that used by certain other investigators. In particular, *first*, we tested: (1) the recordings of GPS (of the high recording frequency of 20 Hz) on an extended range of oscillation frequencies, up to 4 Hz, in order to cover a wider range of civil engineering structures, not only large suspension bridges and high-rise buildings; (2) oscillations with variable frequencies and with up to three different modal frequencies. And *second*, we analyzed our data not only using the traditional FFT technique, but also least-squares-based techniques permitting spectral analysis of discontinuous, non-equidistant or very short time series both in the frequency and in the time domain without prior interpolations or addition of data (zero padding, introducing bias) or fragmentation of time series (leading to loss of information).

The output of this study, as well as some preliminary results of the monitoring of certain bridges indicate that GPS may be used for the structural monitoring of relatively stiff engineering structures, and can identify several modal frequencies up to at least 4 Hz for small (cm-level or even slightly smaller) displacements.

A basic requirement is of course a clear sky-view, i.e. unobstructed view of satellites. Experiments and field studies can be successful only if a large ($\geqslant 5$) number of satellites in favorable constellation are visible. This a priori excludes observations in canyons or ‘‘city canyons’’, as well as observations in high latitudes, since in the latter accuracies in longitude and latitude are essentially different [17].

2. Methodology

The methodology of our study is simple, and basically not much different from what has been used elsewhere [5,9,10,14]. During our experiments we generated horizontal, uni-axial sinusoidal oscillations of known characteristics and recorded the variations of the 3-D coordinates of the moving object with GPS. The resulting time series were analyzed, and computed results were compared with the real (pre-determined) values of the amplitude and of the dominant frequency of the oscillation. Experiments were repeated several times for the same and for various combinations of the oscillation characteristics (frequency, amplitude, characteristics of the experiment), but also oscillations with constant or variable frequencies, or oscillations containing transient signals (i.e. events appearing only in some parts of the examined record).

Hence, we could define statistically the *accuracy* of the method to determine amplitude and dominant frequency of oscillations (i.e. how close the estimates are to the corresponding real values) and not only its *precision* (how close are the estimates to their mean value of a certain parameter), as well as the range of oscillations that can be covered (see Ref. [9]).

Our experiments permitted to investigate not only the performance of GPS to oscillations with characteristics (displacement, frequency) stable during the whole interval, but also with variable ones. Furthermore, although the study oscillations were linear, they could control their 3-D kinematics and spectral characteristics, as is analyzed below.

3. Hardware and experiments

Experiments were based on an apparatus permitting computer-controlled oscillations and a pair of dual frequency GPS instruments (TOPCON Javad Legacy E receivers and Legacy-H antennas). The rover receiver was mounted on top of the oscillating part of the apparatus, while the base receiver was operating in a nearby static position, a few tens to a few hundred meters away, and was recording the same satellite signals at the same frequency of 20 Hz.

The experimental apparatus (ECP Systems Model 210a; [18]), schematically shown in Fig. 1, consists of a PC used to define the oscillation characteristics (frequency, amplitude, duration), of a controller converting the digital signal to analog and transferring it to the oscillator and of an oscillator composed by a servo-motor which can generate linear oscillations with pre-determined characteristics (frequency, amplitude, duration). The oscillator includes one to three wagons, sliding on a straight rail, connected each other by springs, with the first wagon connected through a rigid bar with the servo-motor. Thus an oscillator of one-, two- or three-degrees of freedom, respectively, is formed. In our experiments we were confined to one-degree and three-degrees of freedom in oscillation (one and three wagons, respectively), with the GPS antenna mounted on the first or the third wagon, respectively (Fig. 1).

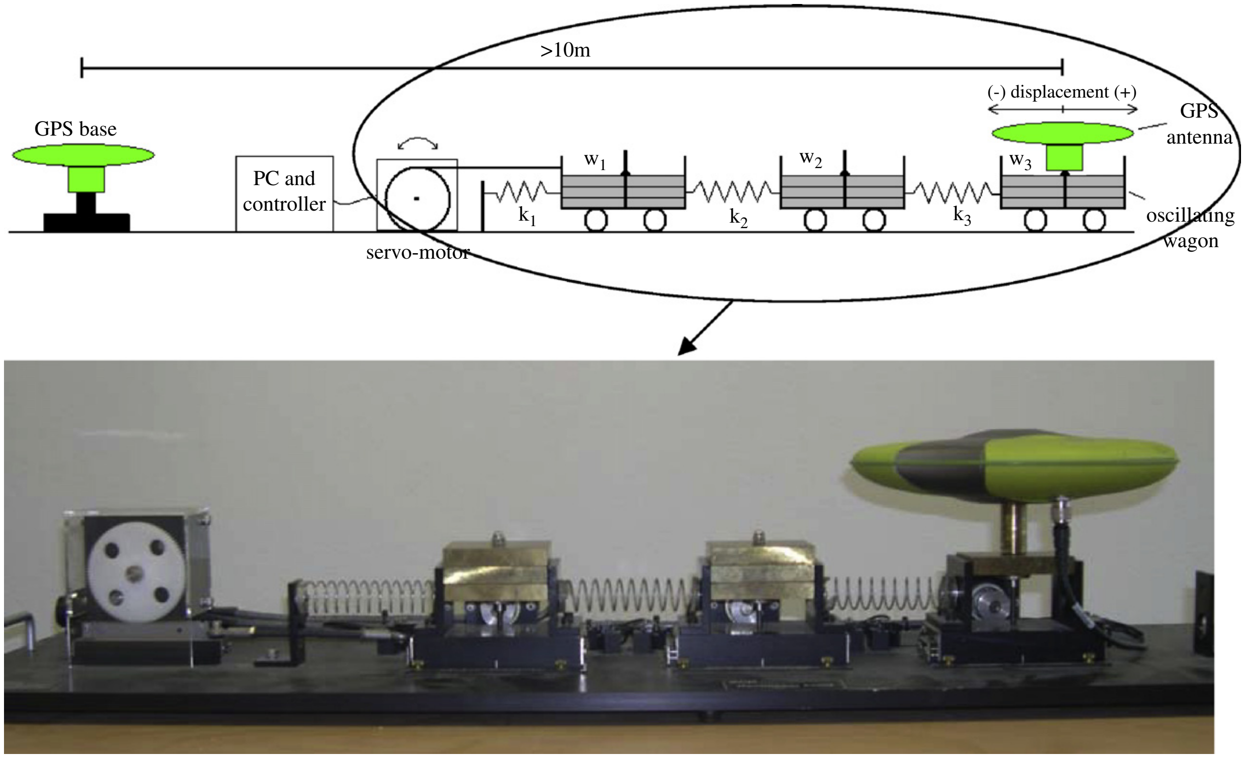

Fig. 1. A diagram and a photo describing the instrumentation used in our experiments permitting three-degrees of freedom. The experimental apparatus consisted of a PC, a controller and an oscillator. The three sliding wagons (W_1, W_2, W_3) are connected with springs (K_1, K_2, K_3) permitting up to three degrees of oscillation. The rover GPS antenna is shown mounted on wagon W_3 and the base GPS receiver on stable ground are also shown. For the cases of experiments of single-degree of freedom, the GPS antenna was mounted on wagon W_1. Part of the apparatus is shown in the photo.

The apparatus used permits to select the mass of each wagon used (simply by setting a different number of metal plates, 0.5 kg each), and use three kinds of springs with different stiffnesses (175, 415 and 770 kN/M). Combining different wagon masses, numbers of wagons and spring types, different modal oscillation frequencies f can be produced. The latter can be calculated from [19]

$$\mathbf{K} - \boldsymbol{\omega}^2 \cdot \mathbf{M} = 0 \tag{1}$$

where **K** and **M** are the matrices of the stiffness and of the mass of the oscillator, respectively, and $\boldsymbol{\omega} = 2\pi/f$. Computed modal frequencies, however, may deviate from real values up to 0.1 Hz, mainly due to uncertainties of the stiffness of the springs.

In our experiments the GPS mounted on the apparatus is subject to a horizontal, linear oscillation which can be analyzed in three parts (Fig. 2c and d):

(1) A middle part covering the major part of the record and corresponding to a harmonic oscillation with constant frequency (excitation frequency) and amplitude, those preset in the experiment (*steady oscillation*).
(2) A short oscillation at the beginning of each record; this oscillation corresponds to the superposition of the harmonic oscillation (seen also in the steady part) and of a rapidly attenuating (damping factor approximately 15%) oscillation lasting for a few cycles only (*transient oscillation*). This transient oscillation is characterized by both the natural frequencies of the oscillator and the excitation frequency and is necessary for the onset of the experiment.
(3) A very short oscillation at the end of the record representing a procedure (“brake”) necessary to stop the oscillation. This third part is ignored in all our further analyses.

In the case of experiments with variable frequency, the oscillation amplitude was also variable, depended on the configuration used (number and mass of wagons, stiffness of springs) and became maximum due to resonance when the oscillation amplitude approaches the modal frequency of the oscillator [19]).

The maximum allowed oscillation amplitude of the device is 3.4 cm (any oscillation with amplitude $\geqslant$3.5 cm is automatically stopped) and oscillation amplitude depends on the oscillation frequency; for frequencies >4 Hz, the corresponding oscillation is limited to 2–4 mm. For these reasons, all our experiments were in the range 0.05–4 Hz in frequency and 0.5–3.4 cm in oscillation amplitude. This range, clearly imposed by the apparatus specifications, satisfies the basic requirements of our study: (1) it exceeds the limit of 1–2 Hz in

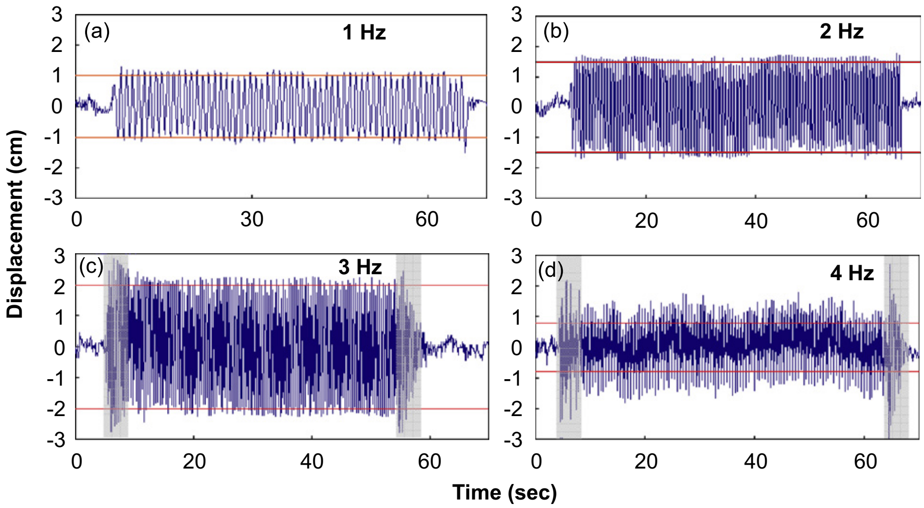

Fig. 2. Displacements for representative experiments with oscillation frequency 1(a), 2(b), 3(c) and 4 Hz (d) used for the spectra of Fig. 4. Gray zones in (c) and (d) indicate the initial and the final part of the time series which correspond to transient oscillations. The middle, main part of the record is characterized by oscillation of constant amplitude and frequency (steady, harmonic oscillation). The red horizontal lines indicate the real oscillation amplitude.

recording frequency of most previous studies; and (2) it covers the oscillation characteristics of most relatively rigid engineering structures, not only long suspension bridges and high-rise buildings. In particular concerning amplitude, the value of 5 mm is the accuracy threshold for the current generation of GPS for kinematic applications [4,12,14] and obviously, larger displacements can easily be detected and analyzed more efficiently.

During our experiments, the antenna of the rover receiver was fixed on an oscillating wagon and the antenna mass was included in the total mass of the wagon. In the case of experiments with one degree of oscillation, only one wagon was connected to the servomotor, while in the case of three-degrees, the GPS receiver was mounted on the third wagon (Fig. 1).

Except for experiments with constant frequency, we made also experiments with linearly increasing frequency, in order to investigate the possibility of GPS to record transient effects and possible changes in the modal frequencies due to seismic damage or soil–structure interactions [20].

More than 250 experiments with an average net duration of one minute, exceeding the duration of most earthquakes, were made between November 2004 and June 2006.

4. Preliminary data processing

GPS records were processed using a commercial software (Pinnacle 1.0) and conventional parameters of the kinematic solution (processing both L1 and L2, 15° elevation mask). Due to the unobstructed view of the horizon by both receivers, the large ($\geqslant 6$) number of the common satellites recorded and the short distance between the two receivers (less than a few hundred meters), permitting near identical signal paths, most common systematic errors (ionosphere effect, etc.) were obliterated, and high accuracy results were obtained. Coordinates were finally computed in the standard WGS84 satellite coordinate system.

At a first step these last coordinates were transformed into a local coordinate system using a standard linear transformation [12,21]. The origin of this new coordinate system was calculated as the mean value of coordinates recorded in each experiment and corresponded to the equilibrium point of the oscillating wagon, while the x-axis corresponded to the oscillation axis. Given that the real movement was linear, displacements along x-axis represented the oscillation signal and those along the y- and z-axis noise of measurements (Fig. 3).

At a second step outliers, not unusual in GPS time series, were cleaned using the 3-sigma criterion [22]. In our experiments very few outliers were identified. Obviously, removal of these outliers produced some gaps in the time series.

Still, outliers are not the only reason for gaps. GPS time series, especially long ones, are usually characterized by gaps resulting: (1) from instrument function interruption (for instance due to power outages); (2) transmission-associated gaps, especially for high ($>$10–20 Hz) frequency signals, mostly due to malfunction of certain long cables connecting antennas with receivers; and (3) signal defects due to poor satellite geometry during a certain time-window and blockage of satellite by obstacles close to the receivers, signal low levels, occasionally signal outages, usually caused by signal interferences (jamming) and secondary reflections (''multipath'' effect), etc. [23]. Such signal defects may frequently be identified and corrected (for instance excluding signals of selected satellites or using filtering techniques), otherwise they lead to gaps in the time series.

5. Spectral analysis

The above problems lead to time series in their majority containing gaps and which cannot be treated with techniques requiring continuous and equidistant data, for instance FFT, the most usual method for spectral analysis. In addition, we had to analyze some very short time series (50–60 points), as will be discussed below.

The usual remedy in such cases is either the division of the available time series into shorter continuous data sets, or the addition of interpolated values or just of zeros to fill the gaps. The disadvantage of these approaches is that they ignore information related to longer period signals, or they introduce additional noise, respectively. For this reason alternative spectral analysis techniques where adopted, the least squares spectrum analysis (LSSA [24]) and the Lomb normalized periodogram (LNP); in particular the Normperiod code [25]. Both techniques are based on the fitting of sinusoidal equations in observations using the least squares method

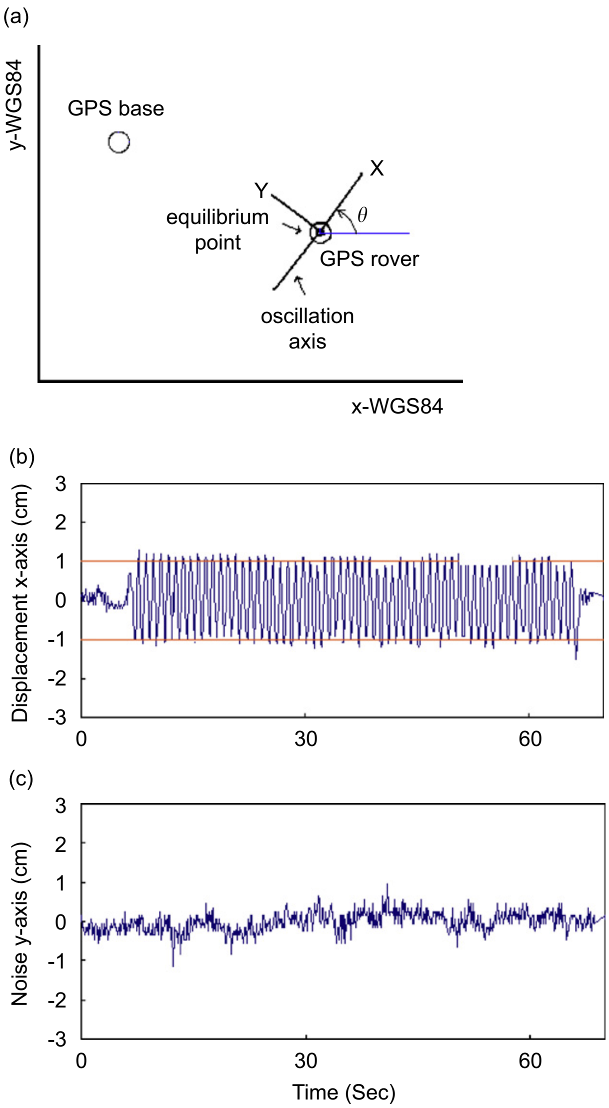

Fig. 3. Transformation of the coordinate system; for simplicity, only the horizontal axes are shown: The initial WGS84 system was rotated by an angle θ and shifted, so that the origin coincides with the equilibrium point of the oscillation (a). For each experiment two time series were obtained: (b) one describing the (real) displacement of the wagon along the oscillation axis (x-axis), and (c) another reflecting the apparent displacement (noise) on a transversal y-axis (down). Horizontal lines determine the real oscillation width. Graph of time series shown corresponds to an excitation frequency of 1 Hz.

(LSQR) and can analyze non-equidistant and discontinuous data. Another advantage of these techniques is that they can provide estimates of the statistical significance of the computed peaks. Furthermore, LSSA can accommodate weighted observations, while Normperiod (or LNP) is simpler (assuming all the data as equally weighted) and hence minimizes computation errors for most data sets [25].

An advantage of both these approaches is that since they are based on least squares techniques they can define the uncertainty level of obtained results. In our calculations we selected the 95% significance level.

In the cases of experiments with variable oscillation characteristics (i.e. oscillation with frequency varying in time), we used the wavelet technique, which can provide a time–frequency analysis [26,27]; in particular, the WINWWZ software [28] which permits the analysis of discontinuous and non-equidistant time series.

6. Results of experiments

6.1. Time series of constant frequency, without gaps, oscillations with single-degree of freedom

A number of experiments with oscillation of single-degree of freedom led to time series without gaps (Fig. 2). Such time series were analyzed basically using FFT. Fig. 4a–d summarizes results for some representative single-degree of oscillation experiments for 1, 2, 3 and 4 Hz. From this figure it is evident that GPS can accurately identify the real oscillation frequency. The time series of some of the experiments were analyzed using the alternative LSSA and LNP (Normperiod) techniques for comparison and results are summarized in Table 1 and Fig. 5. It is evident that computed frequencies with all three techniques differ very little from the real ones (maximum 0.06 Hz), and there is a tendency for the absolute and percent errors to increase and decrease, respectively, at high frequencies (Table 2). The most characteristic effect, however, is the increase in noise (lower precision) expressed mainly as less sharp peaks and secondary peaks in higher frequencies or close to zero ("corner effect" [25]).

6.2. Time series of constant frequency, with gaps, oscillations with single-degree of freedom

Time series with gaps were analyzed using the two known codes based on the Lomb periodogram, LSSA and Normperiod. Fig. 6 shows graphically the results of this analysis for representative time series which corresponded to real oscillation frequencies of 1, 2, 3 and 4 Hz. From this graph it is again evident that the dominant frequency was accurately defined by GPS in all cases, but as was the case with time series without gaps, noise was again increasing with increasing oscillation frequency.

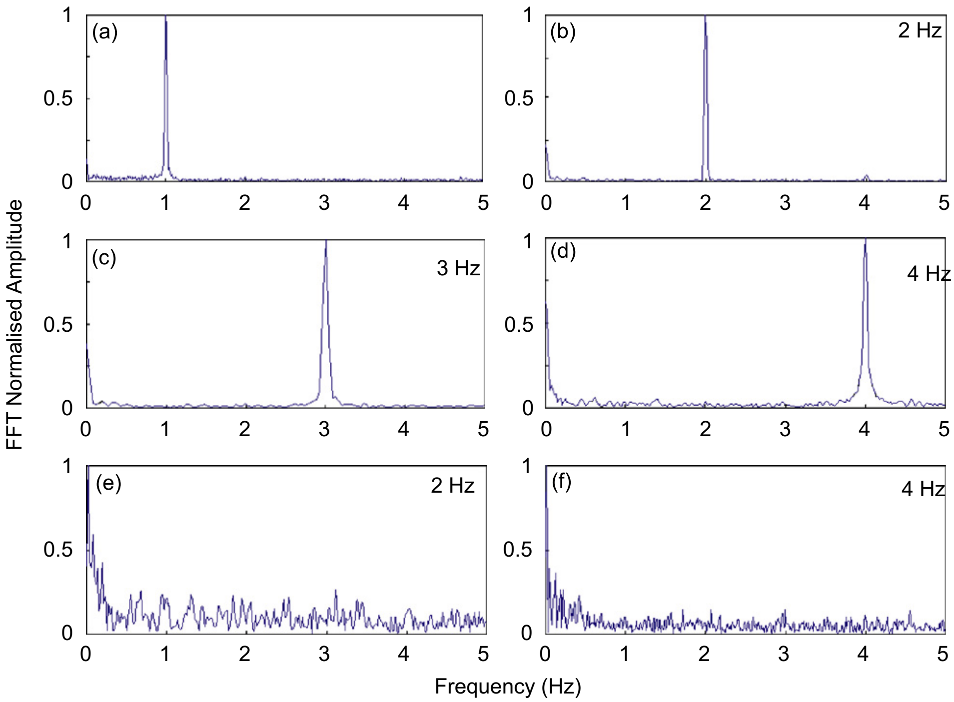

Fig. 4. Results of FFT analysis of time series of representative experiments with oscillation frequencies 1–4 Hz (a)–(d) corresponding to the oscillation x-axis and to the y-axis (lateral to the oscillation axis; (e, f). Increasing width of peaks primarily reflects noise related to decreasing length of time series. Increase in the amplitude of the minor peaks reflects decrease of the signal-to-noise ratio in observations. The FFT analysis of the apparent displacement along the y-axis is characterized by white noise (e, f). Amplitudes are normalized to that of maximum peak in each case. Only the steady part of the oscillation is analyzed (see Fig. 2).

Table 1
Comparison between real and estimated dominant frequencies using FFT, Normperiod and LSSA for time series without gaps, for certain representative experiments

Real oscillation frequency (Hz)	FFT estimate of frequency (Hz)	Typical error (%)	LSSA estimate of frequency (Hz)	Typical error (%)	Normperiod estimate of frequency (Hz)	Typical error (%)
0.05	0.049	2.00	0.048	4.00	0.051	2.00
0.25	0.247	1.20	0.244	2.40	0.248	0.80
0.50	0.495	1.00	0.501	0.20	0.480	4.00
1.00	0.998	0.20	1.006	0.60	0.992	0.80
1.50	1.504	0.27	1.496	0.27	1.490	0.67
2.00	1.991	0.45	1.991	0.45	1.971	1.45
2.50	2.513	0.52	2.493	0.28	2.517	0.68
3.00	3.012	0.40	2.991	0.30	2.984	0.53
3.50	3.513	0.37	3.494	0.17	3.503	0.08
4.00	3.990	0.25	4.060	1.50	4.014	0.35

Typical errors are also shown.

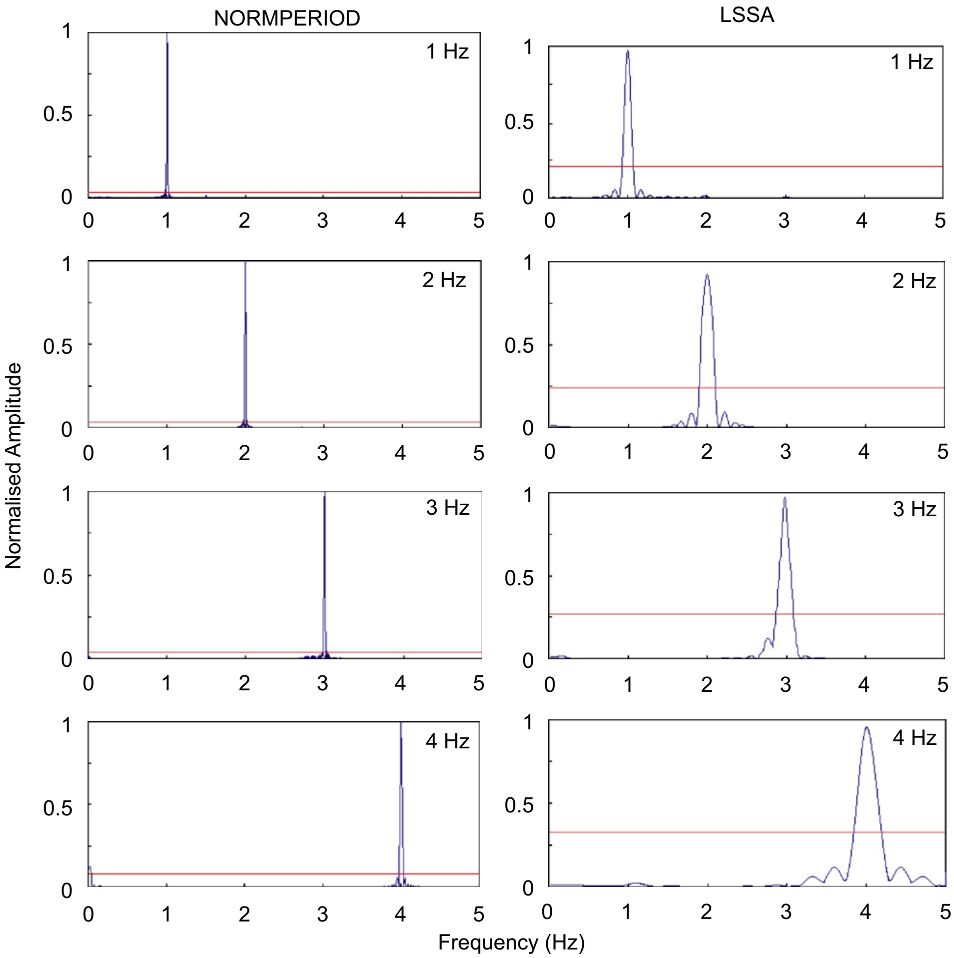

Fig. 5. Results of Normperiod and LSSA analysis of time series representative of experiments with oscillation frequencies 1–4 Hz.

Table 2
Determination of the oscillation frequencies based on LNP analysis of the three-degree of freedom oscillator for oscillation frequencies 1, 2, 3 and 4 Hz. Three cases of each time series were analyzed: the first part of the oscillation (transient oscillation), the sound part (steady oscillation) and the whole time series; see Fig. 2(c,d). Percent differences of determined frequencies from the real ones are also shown. Note that large deviations correspond to very small amplitudes (< 1 cm) and are derived from very short time series)

Excitation frequency			Excitation frequency 1 Hz		Excitation frequency 2 Hz		Excitation frequency 3 Hz		Excitation frequency 4 Hz	
		Real	Computed	Difference (%)	Computed	Difference (%)	Computed	Difference (%)	Computed	Difference (%)
Transient	Excitation	1/2/3/4	0.975	2.5	1.992	0.8	3.008	0.8	3.979	0.5
	Mode 1	0.732	0.731	0.1	0.731	0.1	0.728	0.5	0.776	6.0
	Mode 2	2.332	–	–	–	–	2.329	0.3	2.329	0.1
	Mode 3	3.894	–	–	–	–	–	–	3.688	5.2
Steady	Excitation	1/2/3/4	1.001	0.1	1.999	0.05	3.000	0.0	3.998	0.05
Total	Excitation	1/2/3/4	0.997	0.3	1.999	0.05	3.000	0.0	3.998	0.05

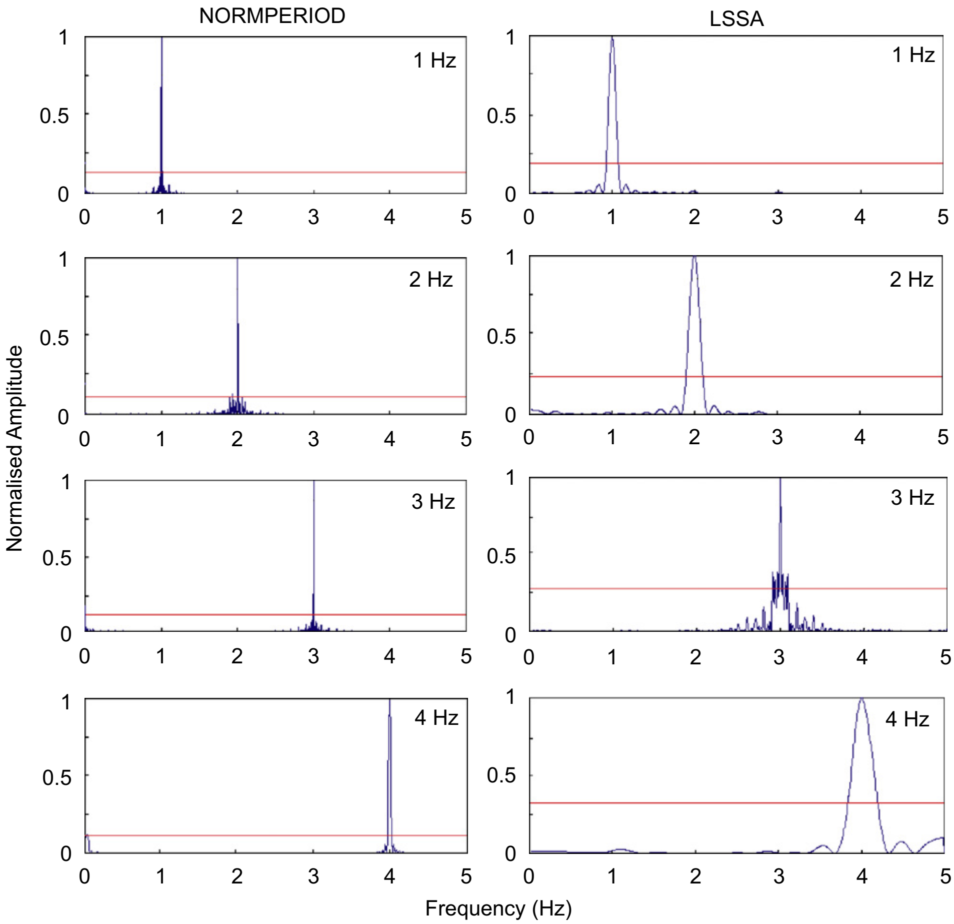

Fig. 6. Results of representative time series analysis containing gaps, for oscillation frequencies 1, 2, 3 and 4 Hz, using the Normperiod and the LSSA code. In all cases the frequency oscillation is accurately defined, but noise (peak width, minor peaks) increases with increasing excitation frequency. Subordinate peaks are mostly due to calculation errors. Amplitudes are normalized to that of the maximum peak in each case. Horizontal lines indicate the 95% confidence level.

6.3. Time series with variable frequencies, oscillations with single-degree of freedom

Time series deriving from experiments with variable frequencies were analyzed in the time–frequency domain using the weighted wavelet Z transform, WWZ based on the Morlet wavelet and the WINWWZ software [26,27]. In these experiments the oscillator was forced to an oscillation with linearly increasing frequency with known characteristics. The spectrogram of this motion is therefore expected to correspond to an inclined straight line delimited by the minimum (initial) and maximum (final) frequency. A characteristic example of the spectrogram of such an experiment for an oscillation frequency increasing linearly from 0.1 to 2 Hz in 120 s is shown in Fig. 7, along with the diagram of the corresponding displacements.

As already noticed, in the case of variable oscillation frequencies the oscillation amplitude is not constant, since it depends on the frequency, and due to resonance, it becomes maximum around the modal frequency of the oscillator [19] (which may be different in each experiment). In the above experiment, near the initial and final points, the amplitude was too small, within the noise limits of GPS, and for this reason no significant

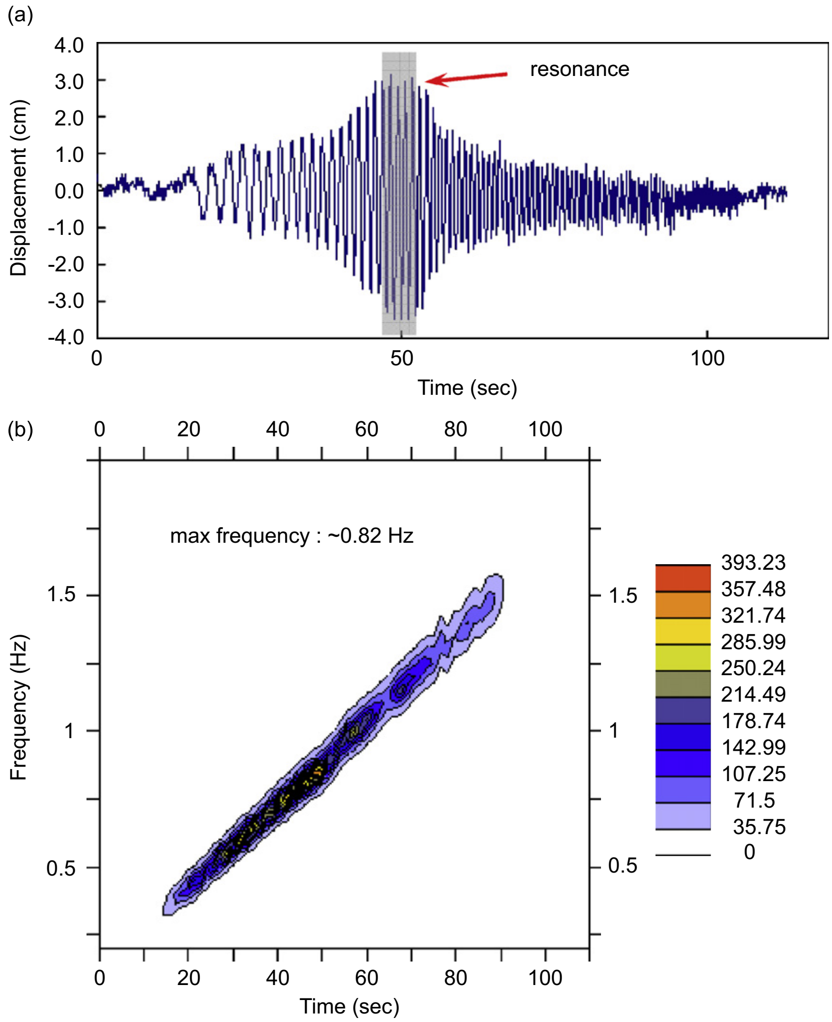

Fig. 7. Recorded displacement (a) and corresponding spectrogram (b) for experiment with a single degree of oscillation and frequency increasing linearly from 0.1 to 2 Hz in 120 s. The spectrogram peak is observed at 0.82 Hz, close to real the modal frequency of the oscillator (0.84 Hz) and correlates with the observed maximum oscillation amplitude (~3 cm at 50 s) due to resonance effects. No significant frequencies were recorded between 0–15 and 90–120 s because oscillation amplitude was too small.

frequency was calculated between 0.1–0.3 and 1.6–2 Hz (Fig. 7). The best determined oscillation frequency was 0.82 Hz, because it was very close to the real (computed) modal frequency of the oscillator, 0.84 Hz.

6.4. Time series with constant frequency, without gaps, oscillations with three-degrees of freedom

A number of experiments with the GPS antenna mounted at the top of the third wagon, i.e. a system with three-degrees of freedom, excited by a constant oscillation were made. Using Eq. (1) and based on the specific configuration of the oscillator (masses of the three wagons and the stiffness of the springs), three modal frequencies were computed, common for all similar experiments (Table 2). The latter mainly led to time series without gaps.

The time series were divided in three parts, corresponding to the first transient and the steady oscillation, only the steady and only the (first) transient oscillation (Fig. 8). For each of these parts and for experiments with oscillation frequencies of 1, 2, 3 and 4 Hz dominant oscillations frequencies computed using the Normperiod code. Fig. 9 and Table 2 summarize results of obtained spectra for a representative experiment.

The first conclusion from Fig. 9 is that in all cases the oscillation frequency was clearly identified from our spectral analysis. For the cases of steady and of steady/transient parts, identification of excitation frequencies was very accurate (maximum difference from the preset one 0.001 Hz). On the contrary, from the analysis of the transient oscillation, this identification was less accurate (maximum difference 0.025 Hz). In these spectra, smaller peaks, below the 95% confidence level, corresponding to the modal frequencies of the oscillator were also detected. These peaks become clear with increasing excitation frequency ($\geqslant 3$ Hz), and their accuracy depends on the corresponding amplitude of oscillation (Table 3). A relatively large difference is observed in the third mode (0.2 Hz), but this is expected because the corresponding amplitude is slightly above the uncertainty error of GPS and time series are too short (50–60 observations). As expected, no significant spectrum could be obtained with time series of this length with FFT.

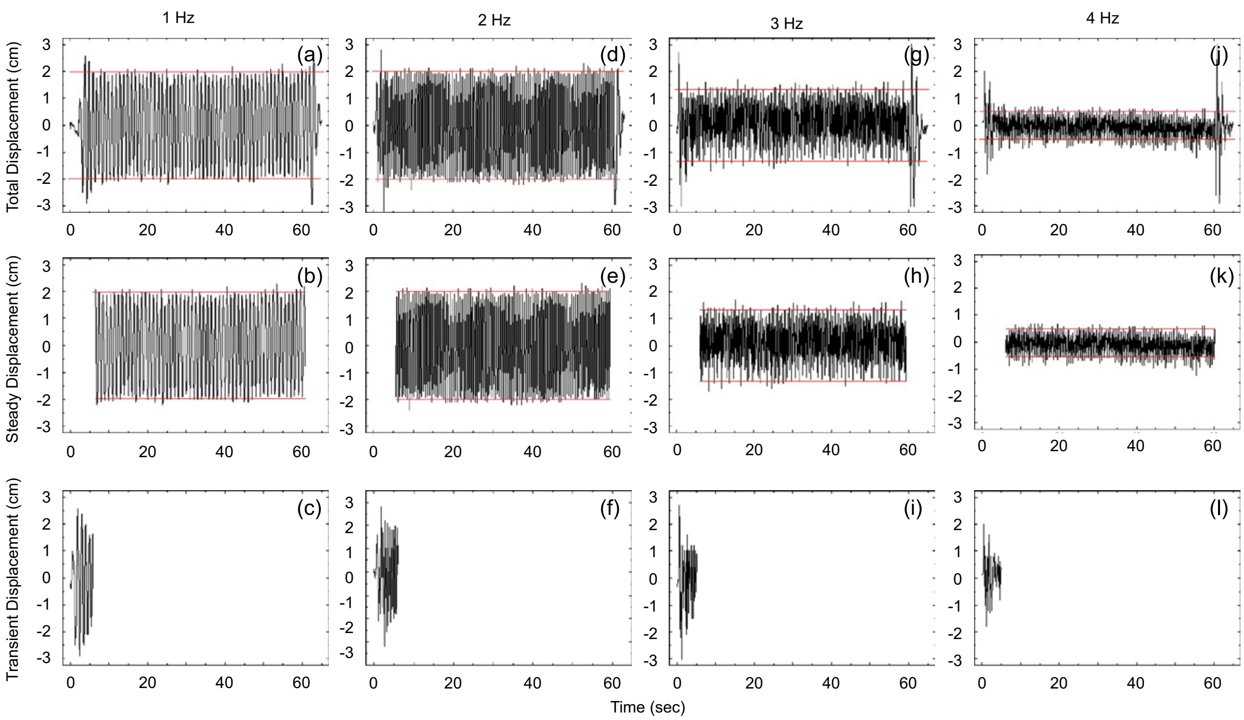

Fig. 8. Representative time series for oscillation frequencies 1 (a, b, c), 2(d, e, f), 3 (g, h, i) and 4 Hz(j, k, l) with three-degrees of freedom. The first row corresponds to the total time series, the second to the steady part and the third row to the first part (transient oscillation). Horizontal lines indicate the real oscillation amplitude.

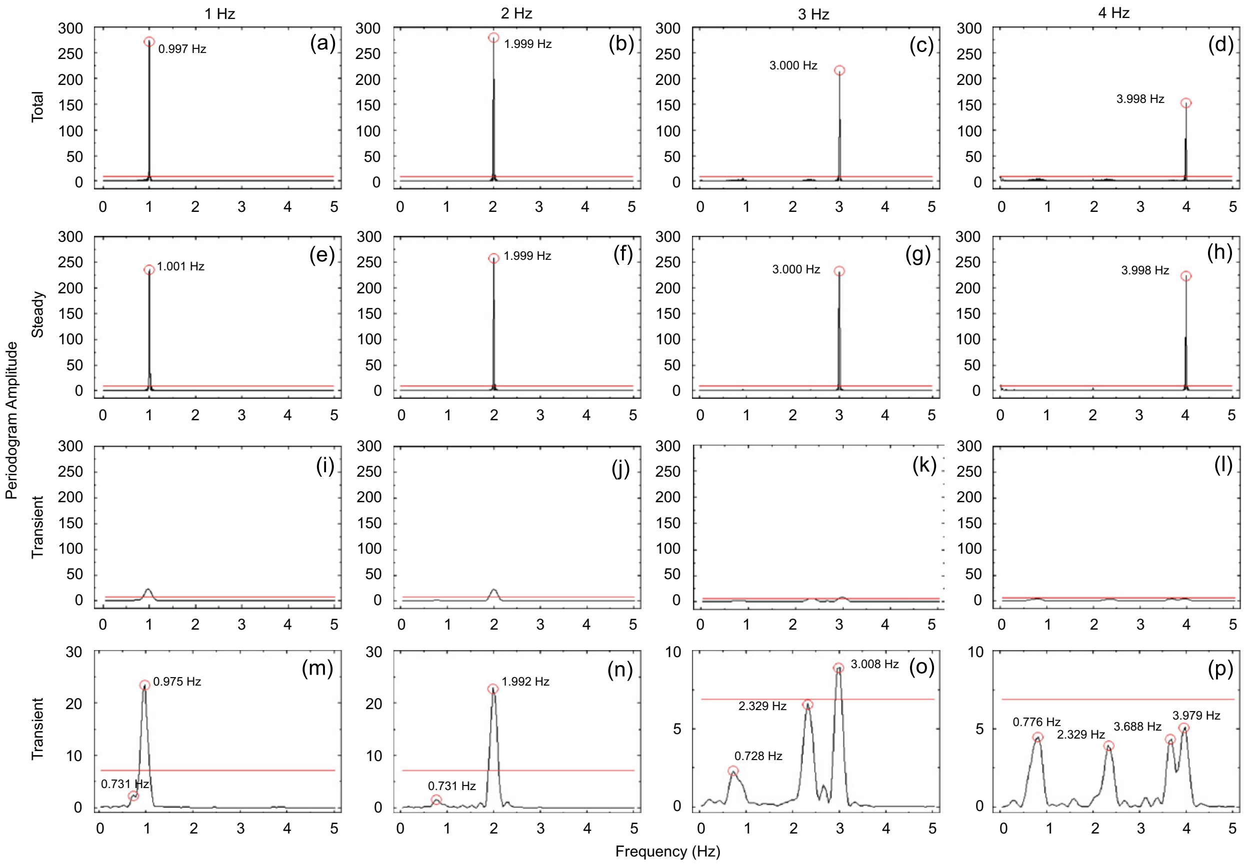

Fig. 9. Spectra of time series of Fig. 8 based on the Normperiod code (oscillations with three-degrees of freedom). Rows correspond to spectra of the first transient and steady (first row; a, b, c, d), steady (second row; e, f, g, h), transient (third row; i, j, k, l) and transient magnified (bottom row; m, n, o, p). Mark that in the first two rows computed excitation frequencies deviate up to 0.009 Hz from the real frequency, but a max deviation of 0.2 Hz is observed for the third modal frequency (compare with Table 3). This deviation is due to the small (8 mm) amplitude corresponding to this mode and to the very short length of the time series used.

Table 3
Modal frequencies computed from a representative example with an experiment with oscillation with three-degrees of freedom

Modal frequency	Computed based on modal analysis	Computed from GPS records	Corresponding oscillation amplitude (cm)
First	0.73	0.79	~3.3
Second	2.33	2.31	~2.9
Third	3.89	3.69	~0.8

(a) Predicted from a modal analysis, the masses of the wagons and the stiffness of the springs; values assumed accurate to ± 0.1 Hz, (b) derived from spectral analysis of the GPS records using the Normperiod code. Approximate amplitude of oscillation is also shown.

6.5. Time series with variable frequencies, oscillations with three-degrees of freedom

The frequency in these experiments was increasing linearly from 0.05 to 4 Hz in 4 min (Fig. 10a). The modal frequencies of the oscillator were computed previously (Table 3). Hence it was expected maximum oscillation amplitude at the vicinity of the first modal frequency and two smaller amplitude peaks at the two other modal frequencies. The GPS displacement record revealed two main peaks correlating with the first two real modal frequencies, and a minor one correlating with the third one (Fig. 10b). The spectrogram of the displacement record obtained using Normperiod code (LNP, analysis in the frequency domain) accurately identified the first

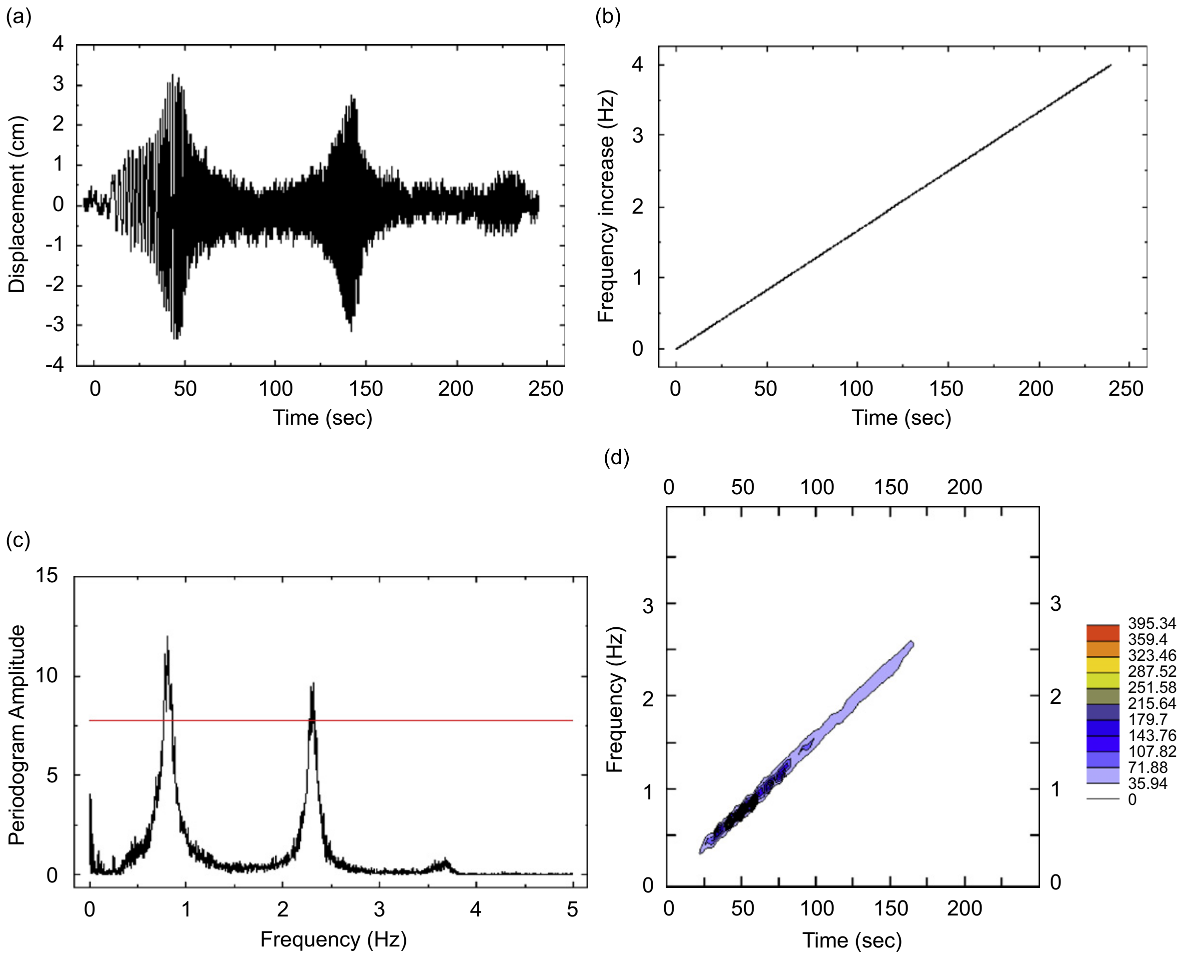

Fig. 10. Results of a representative experiment with three degrees of oscillation (real modal frequencies 0.73, 2.31 and 3.89 Hz) and frequency increasing linearly from 0.05 to 4 Hz in 4 min: (a) displacement; (b) real oscillation frequency; (c) computed LNP (Normperiod) spectrogram (analysis in the frequency domain); and (d) wavelet-based spectrogram (analysis in the time domain). Three displacement peaks are observed in the displacement diagram, indicative of resonance, since they clearly correlate with the real modal frequencies. The two lower modal frequencies are clearly shown in the LNP spectrogram (the third one is shown as a minor peak), but only the first one in the wavelet-based spectrogram, which, however, clearly identifies the frequency changes versus time.

two modal frequencies, as well as the third one, but as a minor peak (Fig. 10c). Analysis in the time domain, on the other hand, using the wavelet technique and the WINWWZ software identified the trend of the frequency changes, but only the lowermost (main) modal frequency (Fig. 10d).

6.6. Time series reflecting white noise

As is shown in Fig. 3, the GPS displacement records were analyzed into the oscillation axis (x-axis) and two axes normal to it (y- and z-axis), which describe only noise of measurements (i.e. apparent movement). Hence, the corresponding amplitude defines the noise level in the data and their spectra should contain only white noise.

This was verified by our analysis; a representative result shown in Fig. 4e and f shows no frequencies significant at the 95% confidence level along the y-axis (the axis normal to the oscillation), in strong contrast to time series reflecting real movement (along the oscillation, or x-axis; Figs. 4 and 5). Furthermore, the amplitude of this time series is very small, practically equal to that before and after the experiment, when the

GPS sensor was still (Fig. 3). It must be noticed, however, that peaks at very low frequencies (<0.15 Hz) indicate systematic errors usually affecting GPS (multipath effect; [7–9,23]).

7. Discussion

The systematic experiments summarized above have two basic characteristics. First, although they are based on uni-axial oscillations, they can define the significance of displacements and spectra obtained in two, or even three axes (compare with Chan et al. [12]). Second, they practically define some thresholds, beyond which results are significant. These thresholds concern both the amplitude of oscillations and the length of the signal. Practically this signifies that if GPS can identify the spectral characteristics of signals with amplitude 5 mm, it can obviously more accurately identify those with amplitude 5 cm or more. Furthermore, longer signals than those examined here (1–4 min) are expected to provide better results, because a better atmospheric model can be produced [5] and the redundancy of the system of observations is increased. The results of our study hence permit to answer the questions discussed in the Introduction.

First, it can be concluded that GPS can identify frequencies of oscillations up to 4 Hz even if the displacement records are noisy (see for instance Figs. 6 and 9). This result is not unexpected, and has been noticed by other investigators as well for lower frequencies [4].

Second, as it can be deduced from Table 1 and Fig. 4, an increase in the frequency of oscillation has little effect in the accuracy in the determination of modal frequencies, but it tends to produce less precise (noisy) results (less sharp peaks; Figs. 4 and 5). This indicates that neighboring peaks may not be distinguished.

Third, several modal frequencies, as well as transient signals in oscillations, can be accurately detected. This confirms previous results of Xu et al. [16] but for long signals and large displacements. A basic limitation, however, is the precision in the determination of each peak: higher frequencies and smaller oscillation amplitudes yield less precise estimates of modal frequencies (less sharp peaks, noise in the spectrum, see Figs. 9 and 10) and hence adjacent modal frequencies cannot be distinguished. The amplitude of oscillation corresponding to each peak is also critical, as Figs. 8 and 10 reveal. Even in the most unfavorable case of multi-degree-of-freedom oscillator excited by a linearly changing frequency in a short period (4 min-long), modal frequencies above 2 Hz corresponding to very small displacements were identified (Table 3).

The basic limitation for such measurements is the requirement for a favorable distribution of satellites being tracked. Clearly, the vertical coordinate is less accurately measured than the horizontal ones [8], but in high-latitude areas the geometry of satellites is poor, and accuracies in the north-coordinate is poor [17]. Of course, significant observations are possible in a clear sky-view, with unobstructed view of at least five satellites at an angle up to 80–85° relative to the zenith. This a priori excludes observations in canyons or "city canyons".

In our analysis we were limited to the minimum filtering of data. Various filtering techniques (for instance Ref. [29]) may improve resolution of results, although in certain circumstances they may introduce some bias.

A final problem is why to use GPS for measuring frequencies of vibration since accelerometers can efficiently do this task? There are several reasons for that: (1) accelerometers, as is the case with all measuring instruments, are affected by errors. Hence, a second, independent type of sensor is very useful. (2) GPS can directly measure displacements, while accelerometer-derived velocities and displacements (see Ref. [30] and references therein cited). In addition, permanent GPS stations in the case of earthquakes can record permanent deformations of buildings [20]. (3) Accelerometers cannot measure low-frequency (semi-static) motions due to their functional characteristics. (4) Accelerometers do not measure time, and hence it is not easy to correlate their recordings; recently indeed, many producers combine GPS in their accelerometers at least for accurate timing of acceleration records. The recent trend, indeed, is the combination of GPS and accelerometer data [6,29,31].

8. Applications of GPS in relatively rigid bridges

Until recently GPS was used in structural monitoring, permitting mainly low (1–2 Hz) modal frequencies of the structures to be revealed [2,5,10,16,32]. In the following, we present some case studies of GPS bridges, high modal frequencies of which were computed: the Gorgopotamos Railway Bridge in Greece and the Wilford Bridge in UK.

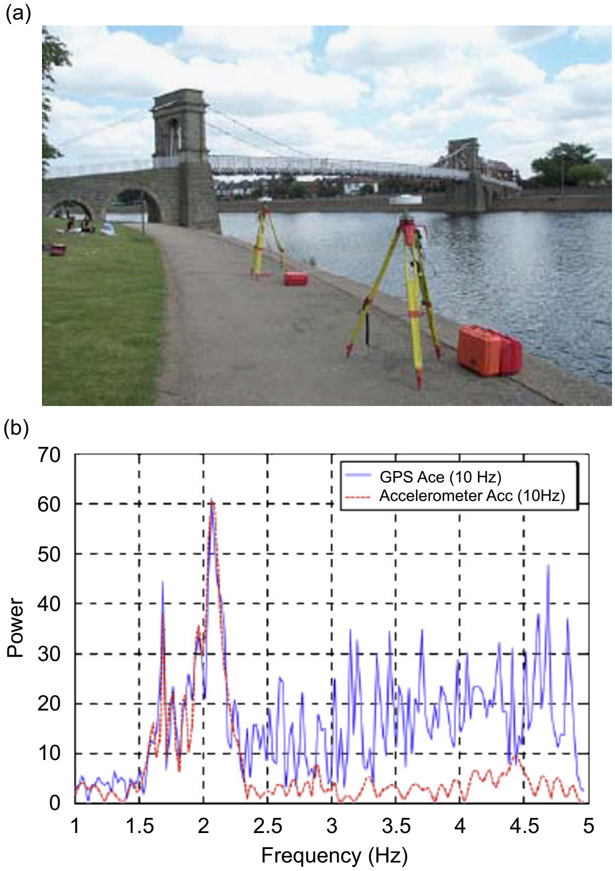

Fig. 11. Accelerometer and GPS spectra (b) of the Wilford Bridge (a), after [29]. Note the close correlations of the peak at 1.733 Hz, corresponding to the 1st modal frequency of the bridge determined by FEM.

8.1. Identification of modal frequencies of Wilford Bridge (Nottingham, UK)

Several tests of excitation of the Wilford suspension footbridge over River Trent in Nottingham were made by IESSG of the Nottingham University [29]. Deflections of the bridge caused by groups of people running or jumping in a pre-determined way and lasting at least 2 min were simultaneously recorded by GPS receivers at 10 Hz sampling frequency and by accelerometers. FFT spectra of the accelerometer and GPS (Fig. 11) were compared with modal frequencies derived from FEM analysis. It was found that GPS had successfully defined modal frequencies in the range >1–2 Hz. This is to the best of our knowledge the only known study of a relatively rigid structure (main span <70 m, modal frequency >1 Hz).

8.2. Identification of modal frequencies of Gorgopotamos Railway Bridge

The Gorgopotamos Bridge in Central Greece is a steel railway bridge consisting of three sets of steel and stone masonry pylons with a spacing of approximately 30 m and the deck is a steel truss (Fig. 12). The central part of the bridge was destroyed and rebuilt twice during the Second World War, making the identification of the modal characteristics of the bridge difficult even by using FEM analysis technique. Thus, the geodetic monitoring of this bridge is expected to produce useful information on its dynamic characteristics.

Based on preliminary results of the GPS records it was observed that the response of the bridge to a passing train is characterized by a vertical oscillation depending on the characteristics of the passing train (number and weight of the passing wagons, etc.). From the spectral analysis of the GPS records based on the Normperiod, a frequency of 0.46 Hz was computed; this probably reflects the excitation frequency of the bridge by a passing

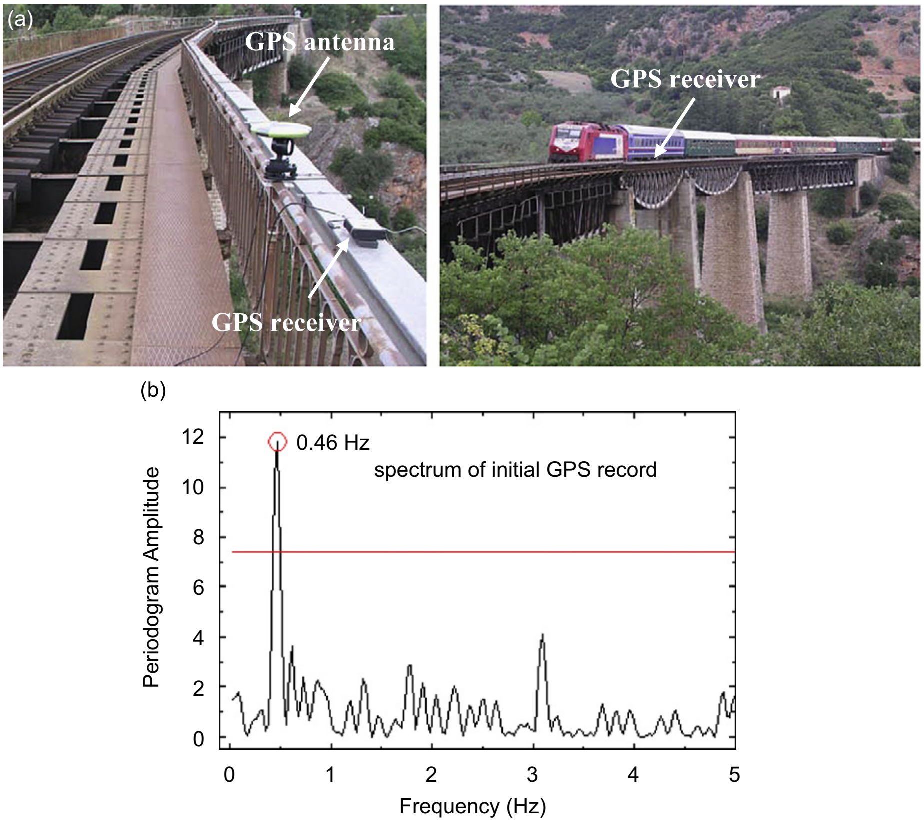

Fig. 12. (a) A GPS receiver recording the response of the Gorgopotamos Train Bridge (Greece) while a train was passing. (b) Spectrum of vertical displacements using the Normperiod code. An excitation frequency of 0.46 Hz and a smaller peak slightly above 3 Hz, reflecting the main modal frequency of the span can be identified.

train. A smaller peak slightly above 3 Hz seems to correspond to the main modal frequency of the bridge (or of the surveyed span). Interestingly, these results are compatible with those derived from independent geodetic data (robotic theodolite [33]) and are obtained for vertical movements which are detected by GPS with accuracy lower than the horizontal ones, but still at a level that permits reliable identification of movements [12] and obviously of oscillation frequencies.

9. Conclusion

The comparison of real (computer-defined) and dominant frequencies computed from the GPS displacement record indicates that GPS can be used to determine spectral characteristics of oscillation in the range of up to 4 Hz using receivers operating at the 20 Hz sampling frequency with excellent results. As far as their accuracy is concerned, results are excellent up to 2 Hz, which corresponds to the main frequencies of dynamic movements of most flexible engineering structures (suspension bridges, pylons etc.), while noise increases at higher frequencies, but without affecting their accuracy. Moreover, it was observed that the accuracy of estimated frequencies does not depend on the oscillation frequency or even the spectral analysis technique used. It was found that analysis of GPS data using wavelet techniques and least squares based software can identify transient signals (i.e. signals not present in the whole part of the time series) if the oscillation amplitude is above the 1 cm threshold, as well as to analyze very short records [33]. Furthermore, experiments with oscillations with three-degrees of freedom, the first reported so far, confirmed that GPS can identify multiple frequencies even in cases of low frequency signal level due to very small oscillation amplitude and short records.

Another conclusion of this study is that, although GPS displacement records may be noisy and characterized by gaps, their spectral characteristics can be defined with high accuracy (Fig. 6). This is due first to the fact that spectra are deduced from a very high redundancy system of observations, and second, that least squares-based spectral analysis techniques, can accurately and easily analyze short or discontinuous time series.

The above lead to the conclusion that GPS is suitable for monitoring a wide range of engineering structures and that it can provide very useful information for their dynamic behavior [9,10,16,34–36] and for the analysis of seismic ground movements [37].

In conclusion, GPS can provide information concerning static, quasi-static and dynamic displacements and oscillation frequencies of a wide range of engineering structures, and hence it can be classified as a promising tool in studies of identification of their real dynamic characteristics and of their transient (for instance due to soil–structure interaction) or permanent (due to damage or alterations) changes, of assessment of their structural health and integrity, and especially in view of modern trends for deformation or displacement-controlled earthquake-resistant design [38,39].

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